Entanglement of Electrons Field-Emitted from a Superconductor

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Under appropriate circumstances the electrons emitted from a superconducting tip can be entangled. We analyze these nonlocal correlations by studying the coincidences of the field-emitted electrons and show that electrons emitted in opposite directions violate Bell's inequality. We scrutinize the interplay between the bosonic nature of Cooper pairs and the fermionic nature of electrons. We further discuss the feasibility of our analysis in the light of present experimental capabilities.

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Entanglement, at the heart of the foundations of quantum mechanics, has gained renewed attention with the birth of quantum information science [1]. Here it is considered to be a precious resource, as it is believed to be the key ingredient for the increased efficiency of quantum protocols as compared with their classical counterparts. Finding sources of entangled particles is therefore of paramount importance. In quantum optics this is well known and routinely used. For example, in parametric down conversion [2] a pump laser beam incident on a nonlinear crystal leads to the generation of entangled photon pairs. In electronics, the field is much younger, but there are already a number of very interesting proposals to generate entangled electron states (see the reviews [3]). In this Rapid Communication, we propose an *entangled electron* source *in vacuum*, on the basis of a thorough analysis of the entanglement and the correlations of the electrons field-emitted from a superconductor.

When a bias voltage is applied to a sharp piece of material, a strong electrostatic field is realized at the tip, causing electron emission into vacuum. The ground state of superconductor is a fairly controllable macroscopic quantum state and provides a coherent and monochromatic electron beam via field emission from a superconducting tip, as experimentally shown in [4]. Our analysis will show that electron coincidences in field emission can reveal electron non-local correlations due to pairing in the superconducting tip. Moreover, we shall see that, by orienting the detectors in opposite directions, one can optimize the fraction of entangled electrons in order to perform a test of Bell's inequality.

Field emission thus enables one to study Bell's inequality on electrons in vacuum by means of correlation measurements. Signatures of quantum statistics and of correlation can be unambiguously detected by coincident measurements. After the seminal result by Hanbury Brown and Twiss [5], bunching and antibunching of bosons and fermions have been measured in a series of important experiments [6, 7, 8, 9, 10, 11]. Advanced technology in single-electron detection has made possible the observation of antibunching in field emission [10]. An experiment with a superconducting tip, similar to [4], is what is needed to test our proposal.

We will show that the spectrum of emitted electronics from a superconductor displays a remarkable interplay between positive correlations (bunching), due to the bosonic nature of the Cooper pairs, and negative correlations (antibunching) due to the fermionic nature of the electrons that make up the Cooper pairs. An electron pair exhibiting positive correlation is in a Bell state. The presence of a bunching-like behavior in quantum transport through multiterminal superconductor-normal metal structures has already been pointed out through an analysis of current noise [12, 13]. Superconductors were also shown to be a source of entangled electrons, whose generation and detection in hybrid conductors has been discussed in a number of articles [14]. In particular, we draw attention to the work of Prada and Sols in [14], where the angular distribution of the emitted electrons from a superconductor has been discussed.

Our setup is sketched in Fig. 1. The Hamiltonian in 3D space reads $H=H_S+H_V+H_T$, with ($\hbar=1$)

$$H_S=\sum_{s=\uparrow,\downarrow}\int d^3\boldsymbol{k}\,\omega_k\alpha_{\boldsymbol{k}s}^\dagger\alpha_{\boldsymbol{k}s},\quad \omega_k=\sqrt{\varepsilon_k^2+|\Delta|^2}, \tag{1}$$

$$H_V=\sum_{s=\uparrow,\downarrow}\int d^3\boldsymbol{p}\,\varepsilon_p c_{\boldsymbol{p}s}^\dagger c_{\boldsymbol{p}s},\qquad \varepsilon_p=\frac{p^2}{2m}-\mu, \tag{2}$$

$$H_T=\sum_{s=\uparrow,\downarrow}\int d^3\boldsymbol{p}\int d^3\boldsymbol{k}\,(T_{\boldsymbol{pk}}c_{\boldsymbol{p}s}^\dagger a_{\boldsymbol{k}s}+T_{\boldsymbol{pk}}^*a_{\boldsymbol{k}s}^\dagger c_{\boldsymbol{p}s}), \tag{3}$$

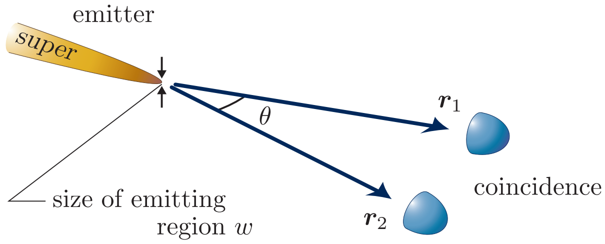

FIG. 1: (Color online) Field emission of electrons and coincident detections. The size of the emitting region at the tip is w and two of the emitted electrons are detected at $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$.

where H_S is the Hamiltonian of the superconducting emitter, $\alpha_{\boldsymbol{k}s}$ the fermionic operators of the quasiparticles, H_V describes the propagation of the electrons in vacuum, $c_{\boldsymbol{p}s}$ are fermionic operators, and the (interaction) Hamiltonian H_T describes the emission of the electrons from the superconductor into vacuum [15, 16, 17]. The energy of an electron in vacuum, ε_p, as well as that of a quasiparticle in the superconductor, ω_k, are both measured from the Fermi level of the superconductor, μ. Δ is the gap of the superconductor and the quasiparticle operators $\alpha_{\boldsymbol{k}s}$ are related to the electron operators $a_{\boldsymbol{k}s}$ by a Bogoliubov transformation [18]. The Coulomb interaction among the emitted electrons can be safely neglected since it becomes relevant at much larger current densities, as compared to those typical of the experiments relevant for the present work [19].

For the tunneling matrix elements we take [17]

$$T_{\boldsymbol{p}\boldsymbol{k}} = h(\boldsymbol{p})g(\boldsymbol{p}-\boldsymbol{k}), \tag{4}$$

$$g(\boldsymbol{p}) = (2\pi)^{-3}e^{-p^2w^2/2}, \quad h(\boldsymbol{p}) = (p/m)^{1/2}e^{\varepsilon_p/2E_C}, \tag{5}$$

where $g(\boldsymbol{p})$ characterizes the emitting region of size w and $h(\boldsymbol{p})$ the tunneling probability, that decays exponentially as the incident energy decreases [20], E_C being an energy scale that characterizes the energy dependence of the tunneling matrix elements.

The emission process is dynamically described in 3D space with the Hamiltonian (1)–(3) and after a transient period it reaches a nonequilibrium steady state [21], with a stationary beam of electrons emitted from the superconductor. Electron correlations are present in this beam [17]. When the detectors do not resolve the spin states of the electrons, the probability of two joint detections, at $(\boldsymbol{r}_1, t_1) = (1)$ and $(\boldsymbol{r}_2, t_2) = (2)$, with $t_2 \geq t_1$, is proportional to

$$\begin{aligned}\rho^{(2)}(2;1) &= \sum_{s_1,s_2=\uparrow,\downarrow} \langle \psi^\dagger_{s_1}(1)\psi^\dagger_{s_2}(2)\psi_{s_2}(2)\psi_{s_1}(1)\rangle \\ &= 4\gamma(2;2)\gamma(1;1) - 2|\gamma(2;1)|^2 + 2|\chi(2;1)|^2,\end{aligned} \tag{6}$$

where $\psi_s(\boldsymbol{r}, t)$ is the field operator of the electrons in vacuum,

$$\gamma(2;1) = \langle \psi^\dagger_\uparrow(2)\psi_\uparrow(1)\rangle = \langle \psi^\dagger_\downarrow(2)\psi_\downarrow(1)\rangle, \tag{7}$$

$$\chi(2;1) = \langle \psi_\uparrow(2)\psi_\downarrow(1)\rangle = -\langle \psi_\downarrow(2)\psi_\uparrow(1)\rangle. \tag{8}$$

The correlation function γ describes the state of single electrons, and, in particular, the (spin-summed) one-particle distribution of the emitted electrons is given by $\rho^{(1)}(\boldsymbol{r}, t) = 2\gamma(\boldsymbol{r}, t; \boldsymbol{r}, t)$. The correlation function χ describes the emission of *pairs* of electrons (with opposite spins). A second-order calculation for the coincident detections at $t_1 = t_2$ yields

$$\chi(2;1) = \int d^3\boldsymbol{k}\, u_k v_k \int \frac{d^3\boldsymbol{p}_1}{\sqrt{(2\pi)^3}} \int \frac{d^3\boldsymbol{p}_2}{\sqrt{(2\pi)^3}} \frac{T_{\boldsymbol{p}_1\boldsymbol{k}}T_{\boldsymbol{p}_2(-\boldsymbol{k})}}{\varepsilon_{p_1}+\varepsilon_{p_2}-i0^+}\left(\frac{1}{\varepsilon_{p_1}+\omega_k-i0^+}+\frac{1}{\varepsilon_{p_2}+\omega_k-i0^+}\right)e^{i\boldsymbol{p}_1\cdot\boldsymbol{r}_1}e^{i\boldsymbol{p}_2\cdot\boldsymbol{r}_2}, \tag{9}$$

where u_k and v_k are the Bogoliubov amplitudes. Notice that, since $u_kv_k = \Delta/2\omega_k$, χ is proportional to the gap parameter Δ and vanishes when the emitter is in its normal state. At zero temperature, there is no contribution from the quasiparticle excitations and Eq. (9) is due to Andreev processes. In the absence of this contribution, the second term in (6) reduces the coincidence probability of finding two electrons close to each other within a small time delay, exhibiting antibunching. The pair correlation χ, on the other hand, enhances such coincidence probability. This is relevant for the occurrence of positive correlations [12, 13]. We now analyze these effects in greater details.

The normalized coincidence

$$Q(r,\theta) = \frac{\rho^{(2)}(2;1)}{\rho^{(1)}(2)\,\rho^{(1)}(1)} \tag{10}$$

at $t_1 = t_2$, when the detectors are at the same distances $r_1 = r_2 = r$ from the tip, is plotted in Fig. 2 as a function of the angle θ between $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$, for normal and superconducting emitters. Here, $k_F = \sqrt{2m\mu} = 2\pi/\lambda_F$. The effects of superconductivity are manifest: a bunching peak appears at $\theta \simeq \pi$. Its origin is clear from the expression of the Andreev process (9) (γ is negligibly small at $\theta \simeq \pi$). This shows that electrons with opposite momenta $\boldsymbol{k}$ and $-\boldsymbol{k}$ are emitted in pair through a virtual process and propagate with momenta $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ in

vacuum, in approximately opposite directions (with unavoidable diffraction effects governed by the size of the emitting region w). The couple $\boldsymbol{k}$ and $-\boldsymbol{k}$ reflects the Cooper-pair correlation in the emitter. Notice that the integrand of χ in (9) is symmetric under the exchange $\boldsymbol{k} \leftrightarrow -\boldsymbol{k}$. This is because the Cooper pair is in a singlet spin state. This symmetry yields bunching, which is observed in opposite directions.

Bunching is therefore a signature of excess singlet pairs, when the emissions take place in opposite directions. It is then of great interest to discuss the nonlocal aspects of the phenomenon [3, 14]. The spin state ϱ of the pair of emitted electrons is

$$\varrho \propto \left(\gamma(2;2)\gamma(1;1) - |\gamma(2;1)|^2\right) \mathbb{1} + 2\left(|\gamma(2;1)|^2 + |\chi(2;1)|^2\right) |\Psi^-\rangle\langle\Psi^-|, \tag{11}$$

where $|\Psi^-\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$ is the singlet state [the normalization factor is given by the two-particle distribution $\rho^{(2)}(2;1)$ in (6)]. Therefore, in general the entanglement of the singlet component is masked by the background.

The degree of entanglement is related to the height $\delta Q = Q(r,\theta) - 1$ of the bunching peak at $\theta = \pi$, which, for $k_F r \gg 1$, $\mu \gg E_C$, and $r/k_F w^2 \gg 1$, is given by

$$\delta Q \sim \frac{\pi^2}{32 K_1^2(|\Delta|/E_C)} \left| H_0^{(2)}\left(\frac{iw^2}{\pi^2\xi^2} - \frac{r}{2\pi^2 k_F \xi^2}\right) - \frac{4\Lambda e^{ir/2\pi^2 k_F \xi^2}}{\pi\sqrt{ir/k_F w^2}} \right|^2, \tag{12}$$

where $K_\nu(z)$ is the modified Bessel function of the second kind, $H_\nu^{(2)}(z)$ is the Hankel function of the second kind, $\xi = k_F/\pi m|\Delta|$ is Pippard's length, characterizing the correlation length of the superconductor, and Λ is a smooth bounded function of w, such that $\Lambda \simeq 1$ for $w \gtrsim \lambda_F$. The higher the bunching peak, the larger the entanglement, and eventually Bell's inequality can be violated. Notice that the electrons of each pair are emitted in opposite directions and one need not argue how to separate them.

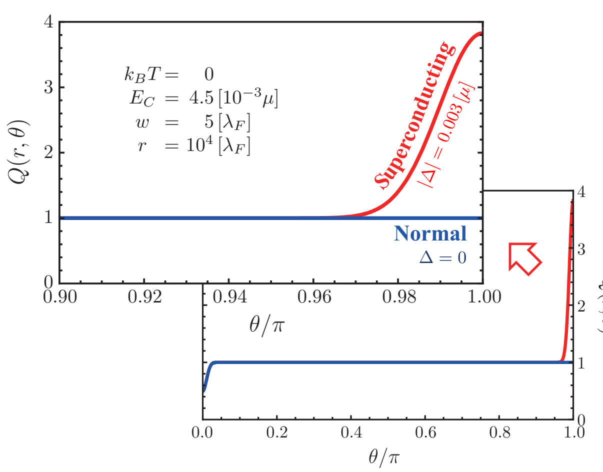

FIG. 2: (Color online) Normalized coincidences $Q(r,\theta)$ vs θ, for normal $\Delta = 0$ and superconducting $\Delta \neq 0$ emitters. Observe the large bunching peak at $\theta \simeq \pi$.

Figure 3 displays the behavior of the bunching peak, by scrutinizing the role played by the parameters describing the system. Clearly, the value of Δ is very significant for entanglement, as the effects of superconductivity are enhanced. By increasing $|\Delta|$, the gap becomes wider, and entanglement is enhanced. The parameter E_C appearing in (5) works like a filter: by decreasing E_C, the contribution of single-particle emission is suppressed, the background is reduced, pair emissions become dominant and entanglement is enhanced.

The effects of w on entanglement are interesting to discuss. Electron pairs emitted from a smaller region bunch better and are more entangled. If the emitting region is larger, there is less guarantee that coincidence electrons originate from a common Cooper pair, and as a consequence entanglement is reduced. This explains the role of the ratio between the size of the emitting region and the extension of a Cooper pair w/ξ, appearing in the formula for the bunching peak (12) and governing the entanglement of the emitted pairs.

Finally, let us focus on the effects of propagation. A smaller value of r yields more entanglement. This is because the wave packets of the emitted electrons spread as they propagate. Even if two electrons are detected at the same distance in opposite directions, this does not ensure that the two electrons originate from a common Cooper pair: there is an ambiguity to the extent of the

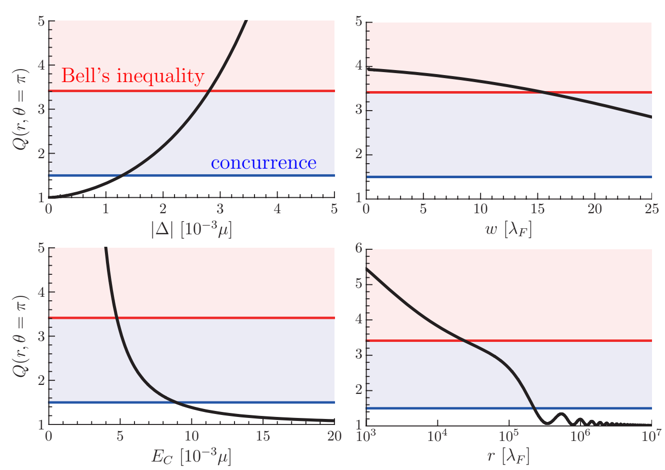

FIG. 3: (Color online) Peak value of $Q(r,\theta)$ at $\theta = \pi$. The line at $Q = 3/2$ indicates the entanglement threshold (above which the pair of electrons is entangled) and the one at $Q = \sqrt{2}/(\sqrt{2}-1) \simeq 3.41$ is that for the violation of Bell's inequality (above which Bell's inequality is violated). The parameters are the same as in Fig. 2, and $\xi \simeq 33.8\,\lambda_F$.

spreads of the wave packets. Due to free-space propagation, the uncertainty at time $t \sim mr/k_F$ is $\lesssim \sqrt{t/m}$ and this value should be smaller than ξ for the two electrons to bunch. The bunching peak (12) actually decays like $\sim k_F\xi^2/r$ for $r \gg k_F\xi^2$ (with oscillation), but the length scale $k_F\xi^2$ is much longer than the extension of a Cooper pair ξ, and the slow decay r^{-1} reflects the divergence in the quasiparticle spectrum. The oscillations of δQ (below the entanglement threshold) shown in the last panel of Fig. 3 for large values of r, are due to the asymptotic behavior of the Hankel function $H_0^{(2)}$.

It is important to check to which extent our results are robust in a non-ideal situation. To this end we analyzed both static fluctuations of the diameter w and the position $\boldsymbol{r}_0$ of the emitting tip. In particular, fluctuations are important only when they become of order of ξ. Moreover, one can show that the angular dependence of the peak is $\propto \exp\{-8k_F^2w^2\sin^2[(\pi-\theta)/4]\}$. Therefore, the effect should be visible as far as $k_Fw\,\delta\theta \lesssim 1$, where $\delta\theta$ is the angle deviation from π in the emission of the two correlated electrons due to local imperfections of the tip. This implies a maximum tolerable value of the roughness of the order of $1/\delta k = 1/k_F\,\delta\theta \simeq w$.

In conclusion we have shown that field emission from a superconducting tip can provide a source of entangled electrons in vacuum. Besides being of great importance for the generation of entanglement in electronics, we believe that a test of Bell's inequality on field-emitted electrons is of interest in itself. Moreover, this would be a remarkable example in which the interplay between the bosonic nature of Cooper pairs and the fermionic nature of electrons is brought to light. Although all the ingredients to experimentally observe our predictions are already available, our analysis shows that stringent requirements should be met, as suggested by Fig. 3. A large energy resolution and a tip material with a large value of the gap are certainly desirable. Also, energy selection close to the Fermi level would enhance correlations.

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of [4], is still small enough to neglect Coulomb effects.

[20] The tunneling probability should not be a function of ε_p but rather of the energy associated with the normal motion to the potential surface. In the following discussion, however, we shall look at far field $k_F r \gg 1$, for which this setting is valid.

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